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SAMPLE SIZES FOR TIMBER CRUISES

by

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Point sampling is now being used for about half of all timber cruises on National Forests in Oregon and Washington. The usual point sampling procedure is to establish a systematic grid of points over the entire timber sale area, to count trees at all points, and to measure all trees at some fraction of all points.

In preparing for a cruise of this kind, decisions must be made on how many points to take for the large sample of "count" points and for the smaller subsample of "measurement" points. These decisions are important, because money and manpower will be wasted if too many points are taken or if the ratio of large-to-small sample sizes is not optimum In other words, overcruising will produce an estimate of total sale volume which is more precise than it really need be, and disproportionate cruising will produce an estimate of total sale volume which is not as precise as it could be for a given cruising cost. If the samples contain too few points, the estimate of total sale volume will be too unreliable for the uses to which it will be put. This report may help the cruiser select proper sample sizes.

Total sale volume was chosen as the critical estimate for determining proper sample sizes, even though total sale volume is not the most important estimate obtained from these timber cruises. The total volume actually sold is normally established by scaling all logs from a sale area after they are cut, and the main purpose of the cruise is to estimate dollar value per unit of volume. However, a sample large enough to produce an acceptable estimate of dollar value per unit of volume will probably not be large enough for an acceptable estimate of total sale volume, and it is for this reason that the estimate of total sale volume becomes critical.

Tables 1 through 4 show the number of measurement points (k) to include in the sample when the amount of sampling error (E) has been specified and when an appropriate index (C_a) to the variation of point volumes around average point volume has been selected.

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Table 1.--Number of measurement points (k), by index of variation (C_a) and sampling error (E), when a measurement point costs three times more than a count-only point (r = 3.00)

Index of variation	Sampling error (standard error of estimated total sale volume expressed as a proportion of estimated total sale volume)							
	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	Number of measurement points							
0.20	200	50	22	12	8	6	-	-
.25	312	78	35	20	12	9	5	-
.30	450	112	50	28	18	12	7	-
.35	612	153	68	38	24	17	10	6
.40	800	200	89	50	32	22	12	8
.45	1,012	253	112	63	40	28	16	10
.50	1,250	312	139	78	50	35	20	12
.55	1,512	378	168	95	60	42	24	15
.60	1,800	450	200	112	72	50	28	18
.65	2,112	528	235	132	84	59	33	21
.70	2,450	612	272	153	98	68	38	24
.75	2,812	703	312	176	112	78	44	28
.80	3,200	800	356	200	128	89	50	32
.85	3,612	903	401	226	144	100	56	36
.90	4,050	1,012	450	253	162	112	63	40
.95	4,512	1,128	501	282	180	125	71	45
1.00	5,000	1,250	556	312	200	139	78	50

NOTE: Multiply the indicated number of measurement points by 3 to find the total number of points.

Table 2.--Number of measurement points (k), by index of variation (C_a) and sampling error (E), when a measurement point costs 5.33 times more than a count-only point (r = 5.33)

Index of variation	Sampling error (standard error of estimated total sale volume expressed as a proportion of estimated total sale volume)							
	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	- - - - - - - - Number of measurement points - - - - - - - - -							
0.20	175	44	19	11	7	5	-	-
.25	273	68	30	17	11	8	-	-
.30	394	98	44	25	16	11	6	-
.35	536	134	60	34	21	15	8	5
.40	700	175	78	44	28	19	11	7
.45	886	222	98	55	35	25	14	9
.50	1,094	273	122	68	44	30	17	11
.55	1,324	331	147	83	53	37	21	13
.60	1,575	394	175	98	63	44	25	16
.65	1,849	462	205	116	74	51	29	18
.70	2,144	536	238	134	86	60	34	21
.75	2,461	615	273	154	98	68	38	25
.80	2,800	700	311	175	112	78	44	28
.85	3,161	790	351	198	126	88	49	32
.90	3,544	886	394	222	142	98	55	35
.95	3,949	987	439	247	158	110	62	39
1.00	4,376	1,094	486	273	175	122	68	44

NOTE: Multiply the indicated number of measurement points by 4 to find the total number of points.

Table 3.--Number of measurement points (k), by index of variation (C_a) and sampling error (E), when a measurement point costs 8.33 times more than a count-only point (r = 8.33)

Index of variation	Sampling error (standard error of estimated total sale volume expressed as a proportion of estimated total sale volume)							
	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	Number of measurement points							
0.20	160	40	18	10	6	-	-	-
.25	250	63	28	16	10	7	-	-
.30	360	90	40	23	14	10	6	-
.35	490	123	54	31	20	14	8	5
.40	640	160	71	40	26	18	10	6
.45	810	203	90	51	32	23	13	8
.50	1,000	250	111	63	40	28	16	10
.55	1,210	303	134	76	48	34	19	12
.60	1,440	360	160	90	58	40	23	14
.65	1,690	423	188	106	68	47	26	17
.70	1,960	490	218	123	78	54	31	20
.75	2,250	563	250	141	90	63	35	23
.80	2,560	640	284	160	102	71	40	26
.85	2,890	723	321	181	116	80	45	29
.90	3,240	810	360	203	130	90	51	32
.95	3,610	903	401	226	144	100	56	36
1.00	4,000	1,000	444	250	160	111	63	40

NOTE: Multiply the indicated number of measurement points by 5 to find the total number of points.

Table 4.--Number of measurement points (k), by index of variation (C_a) and sampling error (E), when a measurement point costs 12.00 times more than a count-only point (r = 12.00)

Index of variation	Sampling error (standard error of estimated total sale volume expressed as a proportion of estimated total sale volume)							
	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10
	Number of measurement points							
0.20	150	37	17	9	6	-	-	-
.25	234	59	26	15	9	7	-	-
.30	337	84	38	21	14	9	5	-
.35	459	115	51	29	18	13	7	5
.40	600	150	67	37	24	17	9	6
.45	759	190	84	47	30	21	12	8
.50	937	234	104	59	38	26	15	9
.55	1,134	284	126	71	45	32	18	11
.60	1,350	337	150	84	54	37	21	14
.65	1,584	396	176	99	63	44	25	16
.70	1,837	459	204	115	74	51	29	18
.75	2,109	527	234	132	84	59	33	21
.80	2,400	600	267	150	96	67	37	24
.85	2,709	677	301	169	108	75	42	27
.90	3,037	759	338	190	122	84	47	30
.95	3,384	846	376	212	135	94	53	34
1.00	3,750	937	417	234	150	104	59	37

NOTE: Multiply the indicated number of measurement points by 6 to find the total number of points.

The total number of points can be found from the relationship between total points and measured points given at the bottom of each table. Thus, from table 1 the number of measured points, k, is 78 when E is 0.04 and when C_a is 0.50. Total number of points (n) is then 3k, or 234.

Each of the four tables is for a particular situation with regard to the relative cost of taking a measured point versus taking a count-only point. If the ratio of point costs is 3.00, table 1 should be used. Tables 2, 3, and 4 are for point cost ratios of 5.33, 8.33, and 12.00, respectively. This range of cost ratios should cover the true cost ratio, whatever it is. These particular cost ratios were selected because they correspond to the integer n/k ratios 3, 4, 5, and 6.

Thus, a cruiser has only to select from these four cost ratios the one he feels is most appropriate. This will automatically establish the ratio of total points to measurement points, and it will also identify the particular table he should use for determining the number of measured points. If none of the four cost ratios is appropriate, new tables can be prepared from equations given in the final section of this report.

Sampling error, or E, in these tables is the standard error of estimated total volume for the entire sale expressed as a proportion of estimated total sale volume. The index of variation, C_a, is the familiar coefficient of variation for point volumes.

In this case,

$$C_a = \frac{F}{\bar{u}_d} \sqrt{\frac{\sum(x_i - \bar{x})^2}{(k-1)}} \quad \text{- - - - - - - - - - - - - (1)}$$

where C_a = coefficient of variation as calculated from a set of sample data

F = basal area factor

x_i = sum of the volume/basal area ratios over all trees at a particular point

$\bar{x}$ = average of the x_i's as calculated from all points in the sample

$\bar{u}_d$ = estimated average volume per acre

k = number of points at which trees were measured for volume.

The accumulated experience from past cruises can be used to help select an appropriate coefficient of variation (C_a) for entering tables 1 through 4. Some information of this kind has been assembled in table 5, but each cruiser will likely have values for C_a which are even more appropriate.

There are actually two coefficients of variation involved in the problem under consideration here. As stated above, one of these is C_a, the index to the variation of point volumes around average point volume. The other is an index to the variation of point volumes around a ratio line of relationship between point volume and point count. It has been given the symbol C_b.

Ideally, advance estimates of C_b should be used along with C_a and E to predict sample sizes. However, all that is presently known about C_b has already been shown in table 5, and this does not offer much encouragement for predicting C_b separately. About all that can be done until more information on C_b becomes available is to assume some fixed relationship between C_a and C_b.

Eighteen of the 23 C_a/C_b ratios in table 5 are clustered closely around 2.0, and the remaining five are considerably higher. Perhaps this justifies the assumption that C_a is normally twice C_b. In any event, tables 1 through 4 were based on this assumption. Later, when more information of the type shown in table 5 is available, it may be possible to identify, in advance, situations which will lead to C_a/C_b ratios other than 2.0. Tables 1 through 4 will not give proper sample sizes for these situations, and new solutions will be required. These new solutions can be developed from the statistical argument presented in the next section of this report.

Table 5.--Planning factors from completed timber cruises for predicting sample sizes in future timber cruises

Cruise No.	National Forest	Timber type[1/]	Average net volume per acre	Basal area factor	Average number trees per point	C_a[2/]	C_b[3/]	C_a/C_b
			M bd. ft.					
1	Gifford Pinchot	DF	46	40.0	5.6	0.64	0.28	2.3
2	Willamette	DF-WH	74	50.0	6.0	.41	.13	3.2
3	Gifford Pinchot	WH-DF	54	38.5	7.8	.38	.08	4.8
4	Siuslaw	DF	53	40.0	4.3	.52	.11	4.7
5	Wenatchee	PSF-DF-WH	52	40.0	6.1	.48	.24	2.0
6	Willamette	DF	72	50.0	6.0	.41	.19	2.2
7	Umpqua	DF	44	40.0	4.2	.64	.33	1.9
8	Wenatchee	PSF	62	40.0	6.0	.44	.22	2.0
9	Rogue River	DF	28	20.0	5.8	.45	.27	1.7
10	Willamette	DF	91	60.0	6.8	.47	.21	2.2
11	Winema	DF-WF	11	20.0	2.3	.97	.28	3.5
12	Wenatchee	PSF-WH	51	40.0	5.6	.55	.22	2.5
13	Willamette	DF-WH	77	60.0	6.3	.41	.20	2.0
14	Winema	PP	6	10.0	3.1	.68	.21	3.2
15	Siuslaw	DF-WH	70	40.0	4.9	.62	.35	1.8
16	Mount Hood	DF	36	40.0	4.9	.60	.28	2.1
17	Mount Baker	WH	50	35.0	6.8	.36	.21	1.7
18	Mount Hood	DF	71	40.0	7.5	.49	.21	2.3
19	Winema	WF	12	20.0	2.9	.68	.33	2.1
20	Siskiyou	DF	42	36.5	5.0	.43	.22	2.0
21	Mount Hood	WF-DF	68	50.0	5.8	.43	.25	1.7
22	Umpqua	DF	31	20.0	6.8	.64	.35	1.8
23	Gifford Pinchot	DF	66	40.0	7.4	.42	.25	1.7

[1/] DF = Douglas-fir, WH = western hemlock, WF = white fir, PP = ponderosa pine, PSF = Pacific silver fir (species are shown in the timber type designation only if they represent 30 percent or more of the total sale volume. Species are listed in order of greatest volume).

[2/] Coefficient of variation for point volumes on measurement points.

[3/] Index to the variation of point volumes around a ratio line of relationship between point volume and point count.

Statistical Argument

Standard error of estimated average volume per acre for the sampling problem under consideration here was shown in a previous publication[1/] to be

$$S_{\bar{u}_d} = F \sqrt{\frac{\sum_{k}(x_i-\bar{x})^2}{n(k-1)} + \frac{\sum_{k}(x_i-Rw_i)^2}{k(k-1)} \left[\frac{n-k}{n}\right]} \quad \text{- - - - - - - - - - (2)}$$

where $R = \frac{\sum_{k} x_i}{\sum_{k} w_i}$

w_i = tree count at any point.

(other symbols were defined earlier in this report)

If equation 2 is divided by estimated average volume per acre and squared, the following result is obtained:

$$E^2 = \frac{(C_a{}^2 - C_b{}^2)}{n} + \frac{C_b{}^2}{k} \quad \text{- (3)}$$

where E = standard error of estimated average volume taken as a proportion of average volume (this, of course, is the same as standard error of estimated <u>total</u> sale volume taken as a proportion of <u>total</u> sale volume).

C_a = See equation 1

$$C_b = \frac{F}{\bar{u}_d} \sqrt{\frac{\sum_{k}(x_i-Rw_i)^2}{(k-1)}} \quad \text{- - - - - - - - - - - - (4)}$$

[1/] Johnson, F. A. Standard error of estimated average timber volume per acre under point sampling when trees are measured for volume on a subsample of all points. U.S. Forest Serv., Pac. NW. Forest & Range Expt. Sta. Res. Note 201, 6 pp. 1961.

Cochran[2/] has shown that E will be a minimum when

$$n = k \frac{\sqrt{(C_a^2 - C_b^2)r}}{C_b} \quad \text{- (5)}$$

where r = average cost of measuring all trees at a point divided by average cost of merely counting trees at a point.

On substituting equation 5 in equation 3,

$$k = \frac{1}{E^2} \left[\frac{(C_a^2 - C_b^2)C_b}{\sqrt{(C_a^2 - C_b^2)r}} + C_b^2 \right] \quad \text{- - - - - - - - - - - - - - - (6)}$$

If this value for k is then used in equation 5 to find n, the sampling effort will be properly apportioned between count points and measured points.

If a fixed C_a/C_b ratio of 2 is assumed, equation 5 reduces to

$$n = k(1.732)\sqrt{r} \quad \text{- (7)}$$

and equation 6 reduces to

$$k = \frac{C_a^2}{E^2} \left[\frac{0.433}{\sqrt{r}} + 0.25 \right] \quad \text{- - - - - - - - - - - - - - - - - - (8)}$$

[2/] Cochran, W. G. Sampling techniques. 413 pp., illus. New York: John Wiley & Sons, Inc. 1963.

Cochran covers this point in his chapter 12 on double sampling. Note that Cochran's Vn is the equivalent of

$$\bar{u}_d^2 C_b^2$$

and that his Vn' is the equivalent of

$$\bar{u}_d^2 (C_a^2 - C_b^2).$$